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INTERFEROMETER INVESTIGATION OF CONVECTION IN A HORIZONTAL FLUID LAYER

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The temperature field in free convective motion of a non-Newtonian fluid is studied by using an interferometer. A method of constructing the flow pattern by means of the interferograms obtained is developed.

An interferometer method is used extensively to visualize temperature fields in gases and liquids [1]. Thus, thermal regime characteristics are determined for a horizontal liquid layer heated from below. A detailed description of the test and analysis methods of the results obtained is presented in [2-4].

The flow pattern in gases and liquids can be observed by using an interferometer only when the velocity changes in the domain under investigation are large, resulting in noticeable density changes (compressibility) and, therefore, in changes in the refractive index also. The velocity gradients in free convection in a horizontal layer are so small that it is impossible to observe the flow pattern by means of an interferometer.

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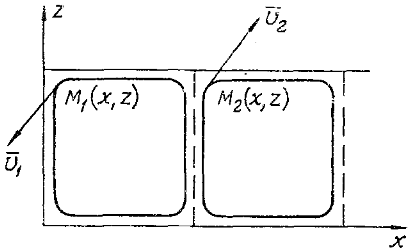

Fig. 1. Schematic portrait of liquid rotation in two adjacent convective points.

Certain types of convective motions do exist which are amenable to visualization, but only by using differential interferometry.

A method of constructing the streamlines and computing the velocity field in free convection in a horizontal layer on the basis of results of measuring shear interferograms is proposed in this paper.

1. Let us write the equation of motion in the Boussinesq approximation

$$\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}=-\frac{1}{\rho_0}\frac{\partial p}{\partial x_i}+\lambda_i g\alpha T+\frac{1}{\rho_0}\frac{\partial \sigma_{ij}}{\partial x_j}. \tag{1}$$

We use the model of an elastic—viscous second-order fluid for the extra-stress tensor

$$\sigma=\varphi_0 A_{ij}^{(1)}+\varphi_1 A_{ik}^{(1)}A_{kj}^{(1)}+\varphi_2 A_{ij}^{(2)},$$

where φ_i are constants of the model, and $A_{ij}^{(1)}$, $A_{ij}^{(2)}$ are Rivlin—Ericsen tensors.

Eliminating $\partial p/\partial x_i$ from (1) by using the operator $\mathrm{curlcurl}_{\theta z}$, then

$$\left(\nu_0\nabla^2-\frac{\partial}{\partial t}\right)\nabla^2 v_z-g\alpha\frac{\partial^2 T}{\partial x^2}=L(M),$$

where

$$L(M)=\frac{\partial^2 M_1}{\partial x\partial z}+\frac{\partial^2 M_2}{\partial y\partial z}-\nabla_1^2 M_3;$$

$$M_i=v_j\frac{\partial v_i}{\partial x_j}-\nu_1\left\{\left(\frac{\partial v_k}{\partial x_i}+\frac{\partial v_i}{\partial x_k}\right)\nabla^2 v_k+\left(\frac{\partial^2 v_k}{\partial x_i\partial x_j}+\frac{\partial^2 v_i}{\partial x_k\partial x_j}\right)\times\right.$$
$$\left.\times\left(\frac{\partial v_k}{\partial x_j}+\frac{\partial v_j}{\partial x_k}\right)\right\}-\nu_2\left\{v_j\frac{\partial}{\partial x_j}\nabla^2 v_i+\frac{\partial v_k}{\partial x_j}\frac{\partial}{\partial x_k}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)+\right. \tag{2}$$
$$\left.+\left(\frac{\partial^2 v_i}{\partial x_k\partial x_j}+\frac{\partial^2 v_k}{\partial x_i\partial x_j}\right)\frac{\partial v_j}{\partial x_k}+\frac{\partial v_i}{\partial x_k}\nabla^2 v_k+\left(\frac{\partial v_k}{\partial x_j}+\frac{\partial v_j}{\partial x_k}\right)\frac{\partial^2 v_i}{\partial x_k\partial x_j}\right\},\ i,\ j,\ k=1,\ 2,\ 3;$$
$$\nu_1=\varphi_1/\rho_0;\ \nu_2=\varphi_2/\rho_0;\ \nabla_1^2=\frac{\partial}{\partial x^2}+\frac{\partial^2}{\partial y^2}.$$

Therefore, L(M) includes the nonlinear characteristics of the motion and the viscoelastic properties of the liquid.

Let the horizontal layer under investigation be bounded by two fixed solid plates, and the temperature as a function of the coordinates be known in advance. Under such hypotheses, it is sufficient to consider (2) with boundary conditions on the horizontal plates to determine the flow pattern.

Convective "rolls" with axes parallel to the horizontal boundaries were detected in our tests with liquids (distilled water, solutions of polyoxyethylene, polyox WSR-301 of different concentrations, etc.).

Because of the repeatability of the elements of the flow pattern along the x axis with a period corresponding to the spacing between adjacent rolls (they rotate in opposite directions), it is sufficient to examine one pair of rolls. The steady-state motion equation in the rolls will have the form

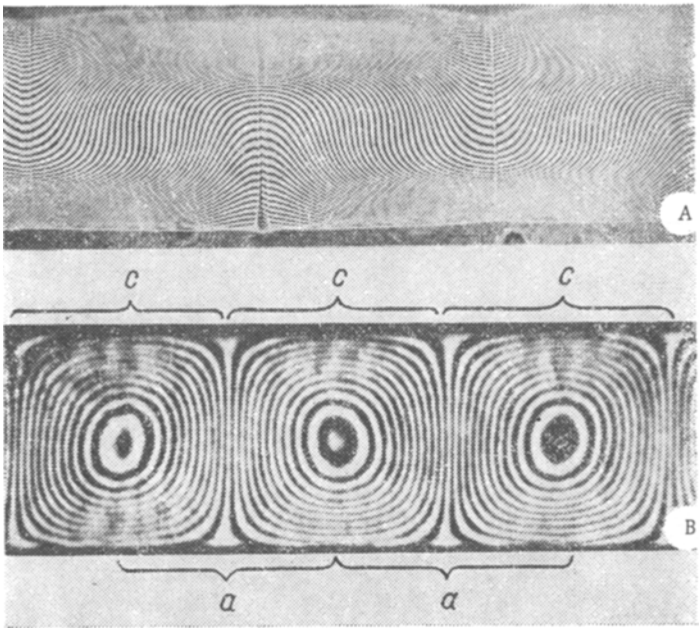

Fig. 2. Interferograms of the temperature field (A) and the field of temperature differences of the rolls (B) in a layer of the water solution of polyox WSR-301 (d = 10 mm, Pr = 340, Ra = 4250, concentration 0.5%).

$$\nu_0 \nabla^2 \nabla^2 v_z - g\alpha \frac{\partial^2 T}{\partial x^2} = L(M). \tag{3}$$

We denote the velocity and temperature in the left roll by $\overline{v}_1(x, z)$ and $T_1(x, z)$ and in the right by $\overline{v}_2(x, z)$ and $T_2(x, z)$. Then for a pair of appropriate points selected arbitrarily (e.g., the points M_1 and M_2 in Fig. 1), on two adjacent rolls, the velocities are equal in magnitude and opposite in direction, i.e., $\overline{v}_2(x, z) = -\overline{v}_1(x, z)$.

If the velocity and temperature fields of the convective roll are superposed on the corresponding field of the adjacent roll, then we describe the resultant motion by the equation

$$2\nu_0 \nabla^2 \nabla^2 v_z = g\alpha \partial^2 \Theta / \partial x^2, \tag{4}$$

where $\Theta(x, z) = T_1(x, z) - T_2(x, z)$ is the horizontal component of the temperature difference at any pair of adjacent rolls.

Equation (4) is valid in the whole domain of convective field superposition. As will be shown below, the described superposition of the fields is determined by using the method of differential interferometry. We write (4) in terms of the stream function $\psi(x, z)$ and the temperature difference $\Theta(x, z)$:

$$2\nu_0 \frac{\partial}{\partial x} \nabla^2 \nabla^2 \psi(x, z) = g\alpha \frac{\partial^2}{\partial x^2} \Theta(x, z). \tag{5}$$

Integrating this last expression along the x axis, we obtain

$$2\nu_0 \nabla^2 \nabla^2 \psi(x, z) = g\alpha \frac{\partial \Theta}{\partial x}(x, z) + C(z), \tag{6}$$

which can be expressed in terms of the vortex $\eta = -\nabla^2 \psi$:

$$2\nu_0 \nabla^2 \eta(x, z) = g\alpha \frac{\partial \Theta}{\partial z}(x, z) + C(z). \tag{7}$$

Let us assume that the vortex lines are rectilinear on the roll boundaries. Therefore, the condition

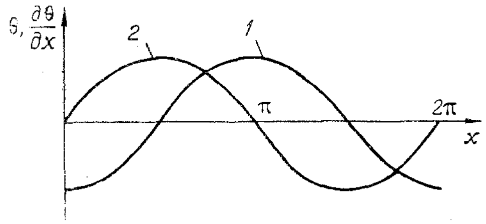

Fig. 3. Graph of the distributions of the function Θ (curve 1) and $\partial\Theta/\partial x$ (curve 2) along the axis OX constructed for z = d/2.

$$\eta|_S = 0 \tag{8}$$

should be satisfied for (7), where S is the boundary of the roll domain. The opposite rotation of the adjacent rollers indicates the presence of points of inflection on the boundaries of the vortex lines, i.e., $\nabla^2\eta|_S = 0$. As will be shown below, $\partial\Theta/\partial x|_S = 0$. Therefore, the function C(z) is zero for any arbitrary section z = const, i.e., everywhere in the domain of the convective roll.

Therefore, (7) and (8) can be written in the form

$$2\nu_0\nabla^2\eta(x,\ z) = g\alpha\,\partial\Theta(x,\ z)/\partial x, \tag{9}$$

$$\eta|_S = 0. \tag{10}$$

The non-Newtonian properties of the liquid under investigation were implicit in the function $\partial\Theta(x, z)/\partial x$, determined from tests by interferograms (Fig. 2).

Having determined the vortex from the boundary-value problem (9) and (10), the stream function distribution $\psi(x, z)$ over the roll can be determined from the problem

$$\nabla^2\psi(x,\ z) = -\,\eta(x,\ z), \tag{11}$$

$$\psi|_S = 0. \tag{12}$$

Therefore, the distribution of the function $\partial\Theta(x, z)/\partial x$ in the plane of the roll, which is determined from interferograms of the fields $\partial\Theta/\partial x$ and Θ obtained on an instrument IAB-451 with the interference adapter RP-452 and the laser LG-75 as light source, must be known to solve the problems (9)-(12).

An interferogram of the temperature field in a viscoelastic liquid layer (polyox WSR-301) is represented in Fig. 2A.

The nature of the isotherm indicates convection of the roll type in the liquid layer. The boundaries between adjacent rolls are seen in the form of vertical dark and light lines. It is interesting to note that these boundaries between the rolls have not yet been observed during convection in Newtonian fluids. The vertical boundaries between adjacent rolls in a non-Newtonian fluid layer can be explained as follows. At the upper (cold) boundary of the layer, the fluid particles in adjacent rolls move opposite to each other on being cooled, and on meeting rotate downward to form a common vertical jet where the temperature is lower than to the right and left of this jet. The next boundary between the adjacent rolls is that recalling a heated jet formed by two merged flows of adjacent rolls proceeding from the lower (hot) boundary of the layer. The macromolecules of the non-Newtonian fluid are oriented in the domains of vertical cold and hot flows because of the maximal velocity gradient on the horizontal boundaries of the layer. Precisely the orientation contributes to the visualization of the vertical boundaries between the adjacent rolls. Their distinctive coloring (dark and light) is explained by the opposing directions of the temperature gradient in the cold and hot vertical jets.

An interferogram obtained during adjustment of the adapter RP-452 by a horizontal shift with a step equal to the roll diameter is presented in Fig. 2B. The periodic structures of the form *a* (Fig. 2B) are therefore the interference pattern of the difference in the temperature fields of each pair of adjacent rolls, i.e., the interferogram of the field Θ. The direction of the interference fringes indicates that $\partial\Theta/\partial x = 0$ on the roll boundaries, as was mentioned above.

Meanwhile, this same interferogram can be considered as a number of periodic structures of the form c (Fig. 2) with the form of closed concentric curves greatly reminiscent of the lines ψ = const or η = const [5].

Let us examine the physical meaning and the relationship between these two structures mentioned above by relying on processing of the interferogram (Fig. 2) for the temperature fields $\Theta(x, z) = T_1 - T_2$ of two adjacent rolls at a certain height z_1. The result of photometry (curve 1 in Fig. 3) indicates that the distribution $\Theta = \Theta(x, z_1)$ can be approximated by the function $k(z_1)\cos x$, $k(z_1) \in [0, 1]$.

By differentiating the function $\Theta = \Theta(x, z_1)$ we construct the dependence on the coordinate x (curve 2 in Fig. 3). As should be expected, the dependence $\partial\Theta/\partial x = \partial\Theta(x, z_1)/\partial x$ is sinusoidal. We obtain this same sinusoid by photometry of a structure of the form of Fig. 2. Therefore, differentiation of the field $\Theta = \Theta(x, z_1)$ with respect to x results in a shift of the periodic structure in Fig. 2 to the left on half the roll. On the other hand, a cosinusoidal nature of the distribution $\Theta = \Theta(x, z_1)$ would permit the assertion that integrating the function $\Theta = \Theta(x, z_1)$ would lead to a shift to the right on half the roll.

Thus, periodic structures of the form a (see Fig. 2) yield the isolines of Θ, and structures of the form c yield the isolines of $\partial\Theta/\partial x = \partial\Theta(x, z_1)/\partial x$.

2. Let us examine the solution of the system (9) and (10), which is a classical mathematical physics problem. Using the known method of the Green's function [6], we obtain a solution of the problem (9), (10) in the form of the integral

$$\eta(\xi, \zeta) = -\frac{\alpha g}{2\nu_0} \iint\limits_{(x,z)} \frac{\partial\Theta}{\partial x} \ln \frac{1}{|\psi(y, \tau)|} dxdz, \tag{13}$$

where $\psi(y, \tau)$ is a function that maps the domain of the convective roll conformally on the unit circle. However, utilization of (13) in engineering calculations is difficult; consequently, we propose the following method to determine the roll-type convective flow pattern.

Let us make Eq. (9) dimensionless by using the following variable: $(\xi, \zeta) = (x\pi/a, z\pi/d)$. Then (10) is reduced to

$$\nabla^2\eta = \frac{R_M}{2}\frac{\partial\Theta}{\partial\xi}, \tag{14}$$

where R_M is the dimensionless parameter

$$R_M = \frac{g\alpha\Theta_{max}}{\varkappa\nu_0}\left(\frac{d}{\pi}\right)^3. \tag{15}$$

For any given boundary conditions the flow can therefore be characterized by one dimensionless parameter R_M. The difference between the parameter R_M and the ordinary Rayleigh number

$$\mathrm{Ra} = \frac{g\alpha\Delta T}{\varkappa\nu_0} d^3 \tag{16}$$

is the replacement of the vertical temperature difference ΔT by the maximal horizontal component of the temperature difference of corresponding points of the convective rolls Θ_{max} and the normalization of the layer thickness to the number π. Let us note that just in the case of the convective regime is R_M different from zero, which indicates the introduction of this dimensionless parameter is well-founded. The introduction of R_M permitted approximating the distribution of Θ' and $\partial\Theta/\partial x$ for $\zeta = \pi/2$ by trigonometric functions of the form (Fig. 4) $\Theta' = \cos\xi$, $\partial\Theta'/\partial\xi = -\sin\xi$. Therefore, by using the results of interferometer investigations of the temperature field, and knowing the cosine nature of the distribution $\Theta(x, z)$, the numerical values of the stream function ψ and the vortex function η can be determined in the region of the convective roll.

The method utilized is suitable for small supercriticalities when the velocity and temperature distribution in the roll domain can be represented by a separate Fourier component, i.e., for $\varepsilon = (\mathrm{Ra} - \mathrm{Ra}_{cr})/\mathrm{Ra}_{cr} \leqslant 3$ ("linear" domain) [7]. For $\varepsilon > 3$ higher harmonics are already of substantial influence.

The distribution of the vertical component of the convective velocity W_{max} is shown in Fig. 4 for a horizontal layer of an aqueous polyox WSR-301 solution, computed by the method elucidated above for the section d/2.

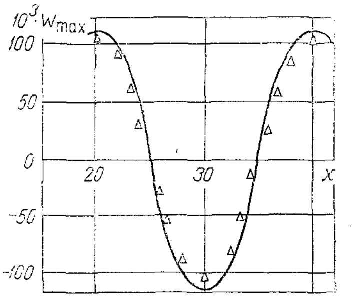

Fig. 4. Distribution of the vertical fluid velocity component computed for the height d/2; x, mm; W_{max}, mm/sec.

NOTATION

α, coefficient of volume expansion, λ_i, (0, 0, 1); g, free-fall acceleration; ρ_o, fluid velocity; v_i, velocity components; p, pressure, T, deviation of the temperature from the mean value; σ_{ij}, tangential stress tensor; $Pr = \nu_o/\varkappa$, Prandtl number; $Ra = (g\alpha/\nu_o\varkappa)\Delta T d^3$, Rayleigh number; ν_o, kinematic viscosity coefficient; $\varkappa$, thermal diffusivity coefficient; ΔT, temperature difference between the layer boundaries; d, layer thickness.

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